

Electrostatics - Electro + Statics

↓
Electric field

↓
Steady or stationary

or non-varying field with time.

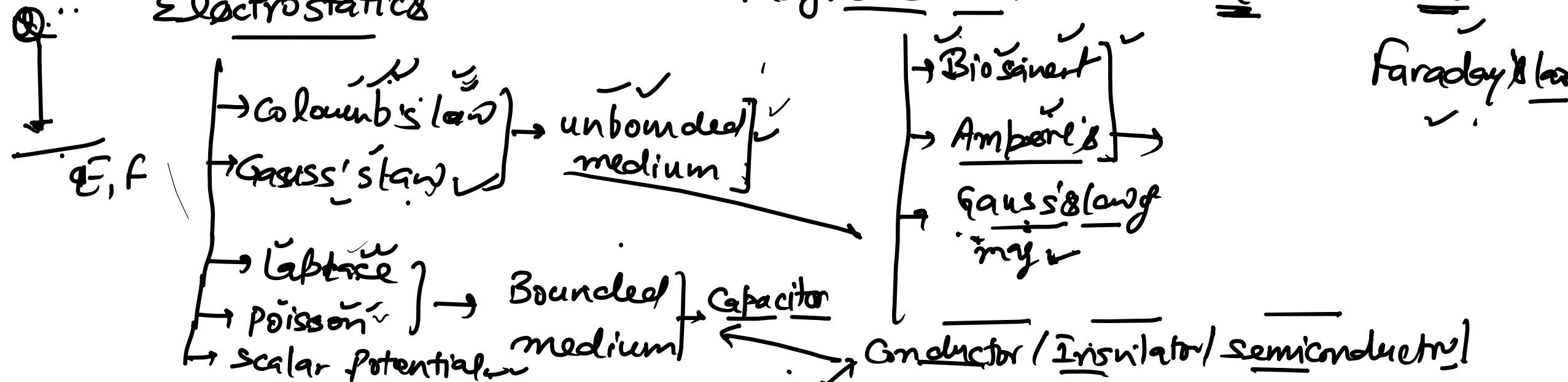
unit - I → Basic tools

Electrostatics

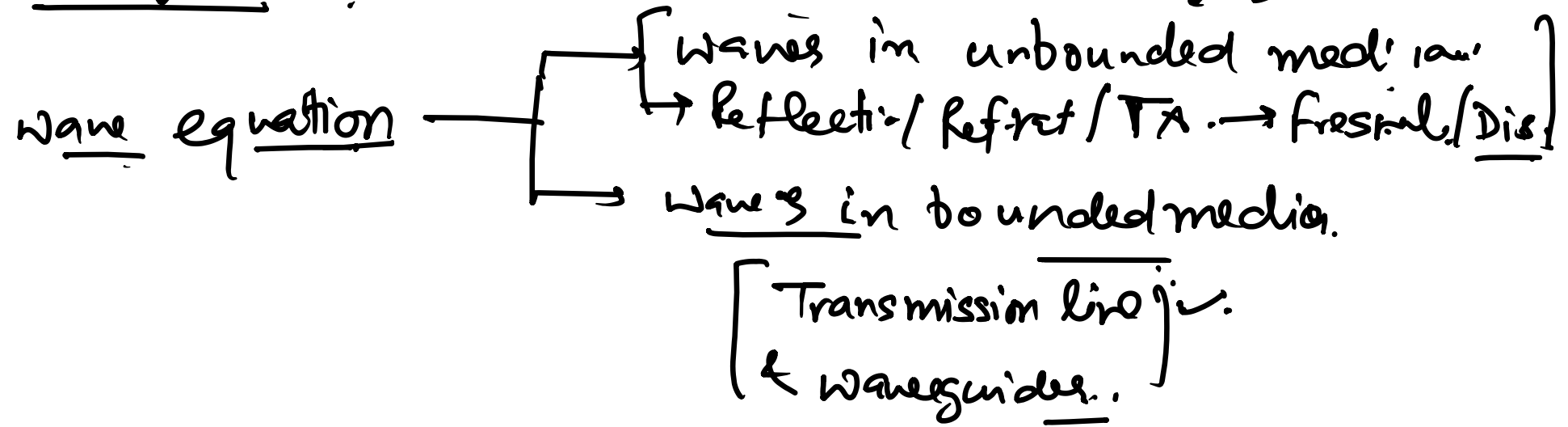
magnetostatics

E_t

H_t
Faraday's law



maxwell's Equations →



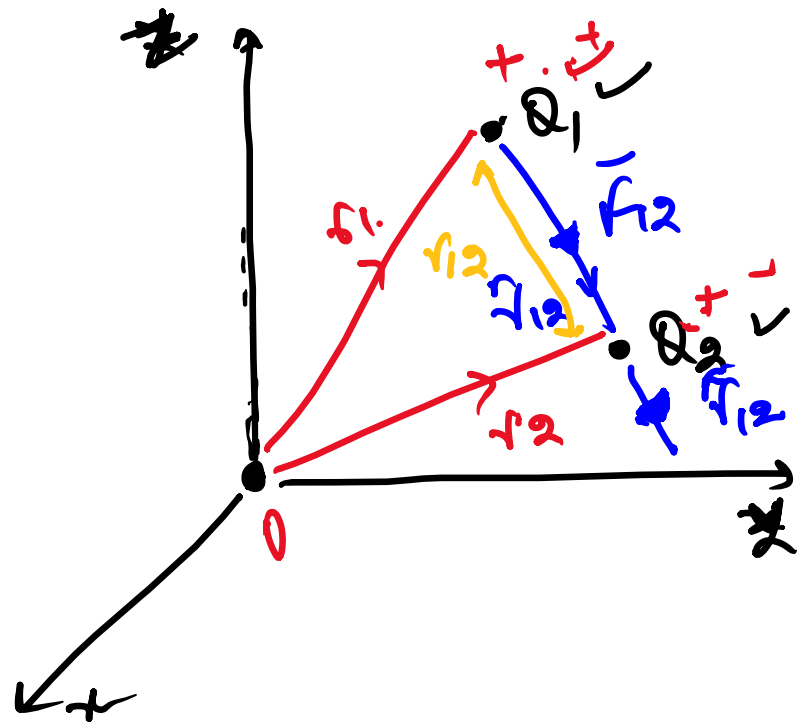
Applications →

[Antennas]

Coulomb's law of forces -

Force on Q_2 due to $Q_1 = \vec{F}_{12}$

Charges - ① Point ✓
 ② Line ✓
 ③ Surface ✓
 ④ Volume ✓



① $\vec{F}_{12} \propto Q_1 Q_2$

② $\vec{F}_{12} \propto \frac{1}{r_{12}^2}$

$\rightarrow \vec{F}_{12} \propto \frac{Q_1 Q_2}{r_{12}^2}$

$\vec{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{r_{12}^2} \hat{r}_{12}$

$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9$

$\vec{r}_{12} = \vec{r}_2 - \vec{r}_1$

$\hat{r}_{12} = \frac{(\vec{r}_2 - \vec{r}_1)}{|\vec{r}_2 - \vec{r}_1|}$

if $r = |\vec{r}_2 - \vec{r}_1|$

$\vec{F}_{12} = \frac{Q_1 Q_2}{4\pi\epsilon_0 |\vec{r}_2 - \vec{r}_1|^2} \frac{(\vec{r}_2 - \vec{r}_1)}{|\vec{r}_2 - \vec{r}_1|}$
 $\vec{F}_{12} = \frac{Q_1 Q_2 (\vec{r}_2 - \vec{r}_1)}{4\pi\epsilon_0 |\vec{r}_2 - \vec{r}_1|^3}$

$$\vec{F}_{12} = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^3} \vec{r}$$

$$\vec{r} = \vec{r}_2 - \vec{r}_1$$

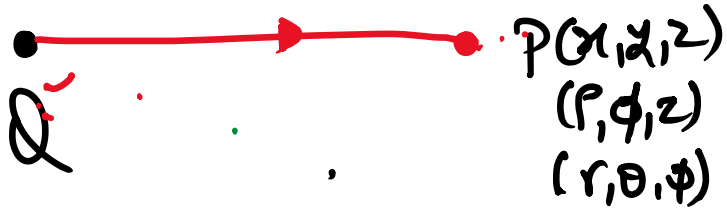
$$r = |\vec{r}| = |\vec{r}_2 - \vec{r}_1|$$

Electric field $\rightarrow$

$$\vec{E} = \lim_{Q \rightarrow 0} \frac{\vec{F}}{Q}$$

$\leftarrow$ DC field

Q solve - Example - 4.1, 4.2, 4.3.



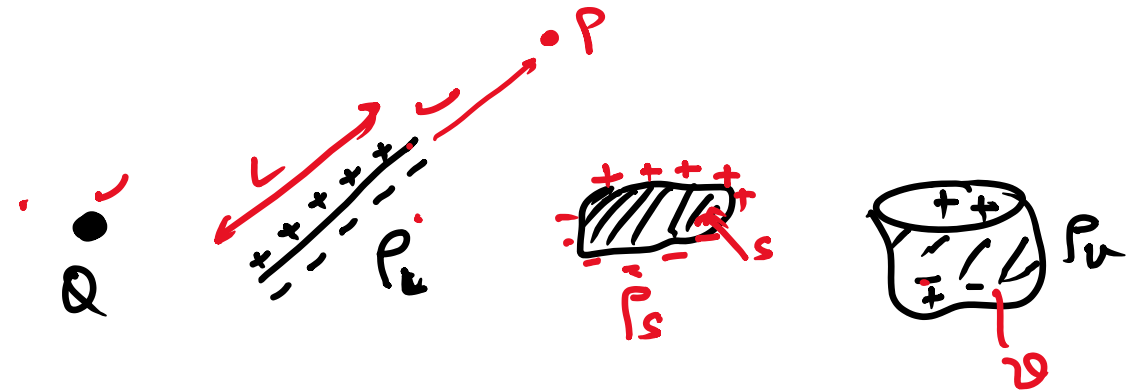
Calculate - $\vec{R}$, $\hat{R}$, $|\vec{R}|$

$\rho_s \rightarrow$ Surface charge density.

$$\rho_s \rightarrow \frac{Q_s}{S} \Rightarrow \rho_s \cdot S = Q_s$$

$$\vec{E} = \frac{\vec{F}}{Q} = \frac{Q}{4\pi\epsilon_0 R^2} \hat{R}$$

- Charges $\rightarrow$
- ① Point charge
 - ② Line charge
 - ③ Surface charge
 - ④ Volume charge



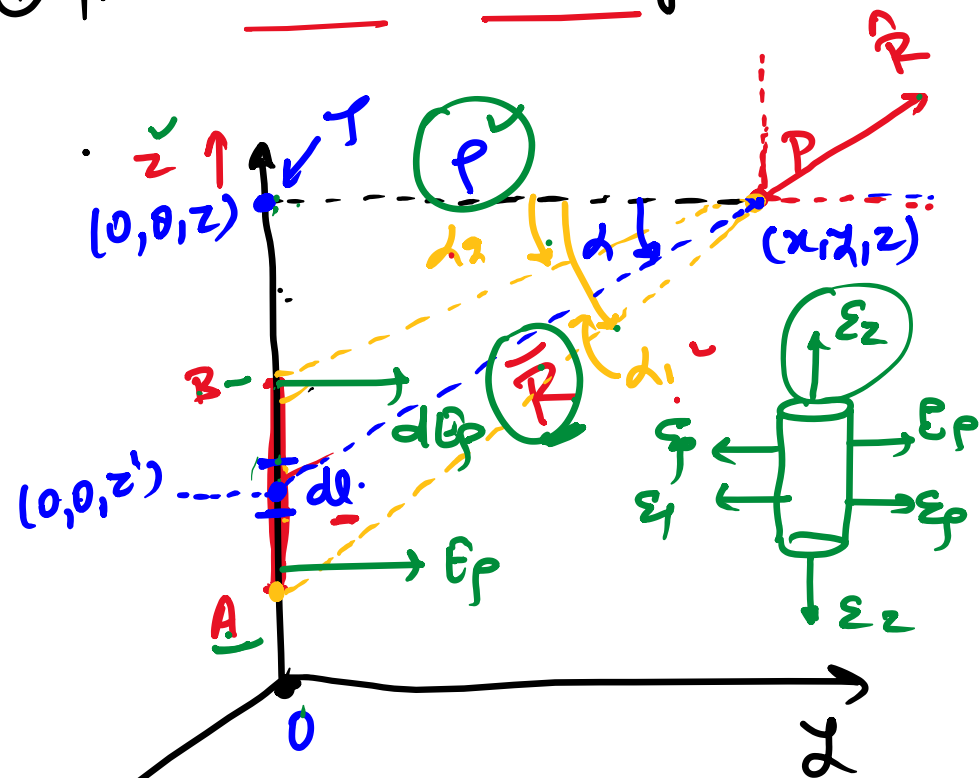
$\rho_L \rightarrow$ line charge density

$$\rho_L = \frac{Q_L}{L} \Rightarrow Q_L = \rho_L \cdot L$$

$\rho_v =$ Volume charge density

$$\rho_v = \frac{Q_v}{V} \Rightarrow Q_v = \rho_v \cdot V$$

① field due to line charge →



$$dl = dz'$$

$$z' = OT - p \tan \alpha$$

$$dz' = -p \sec^2 \alpha d\alpha \quad \text{--- ②}$$

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 R^2} \hat{R}$$

$$Q = \rho_L \cdot L$$

$$Q = \rho_L dl$$

$$d\vec{E} = \frac{\rho_L dl}{4\pi\epsilon_0 R^2} \hat{R} = \frac{\rho_L dl (\vec{R})}{4\pi\epsilon_0 |R|^3} \quad \text{---}$$

$$\hat{R} = \frac{\vec{R}}{|R|}$$

$$\vec{R} = x\hat{x} + y\hat{y} + (z - z')\hat{z}$$

$$(\vec{R}) = \rho \hat{\rho} + (z - z')\hat{z}$$

$$|\vec{R}|^2 = \rho^2 + (z - z')^2 \quad \text{--- ①}$$

$$|R|^2 = \rho^2 \sec^2 \alpha$$

$$\vec{R} = |R| \cos \alpha \hat{\rho} + |R| \sin \alpha \hat{z} \quad \text{--- ③}$$

$$dE = \frac{\rho_L}{4\pi\epsilon_0} \cdot \frac{(-\rho \sec^2 \alpha) d\alpha (\cos \alpha \hat{r} + \sin \alpha \hat{z})}{\rho^3 \sec^3 \alpha} (R)$$

$$dE = \frac{\rho_L}{4\pi\epsilon_0} \frac{(-\cancel{\rho} \sec^2 \alpha) d\alpha (\cos \alpha \hat{r} + \sin \alpha \hat{z})}{\rho^3 \sec^3 \alpha} \cancel{\rho} d\alpha$$

$$dE = \frac{\rho_L}{4\pi\epsilon_0} \left(- \frac{(\cos \alpha \hat{r} + \sin \alpha \hat{z})}{\rho} \right) d\alpha$$

$$\vec{E} = - \frac{\rho_L}{4\pi\epsilon_0} \int_{\alpha_1}^{\alpha_2} \frac{(\cos \alpha \hat{r} + \sin \alpha \hat{z})}{\rho} d\alpha$$

Thus we can calculate the electric field due to a line charge of finite length.

→ Case II → Infinite line charge -

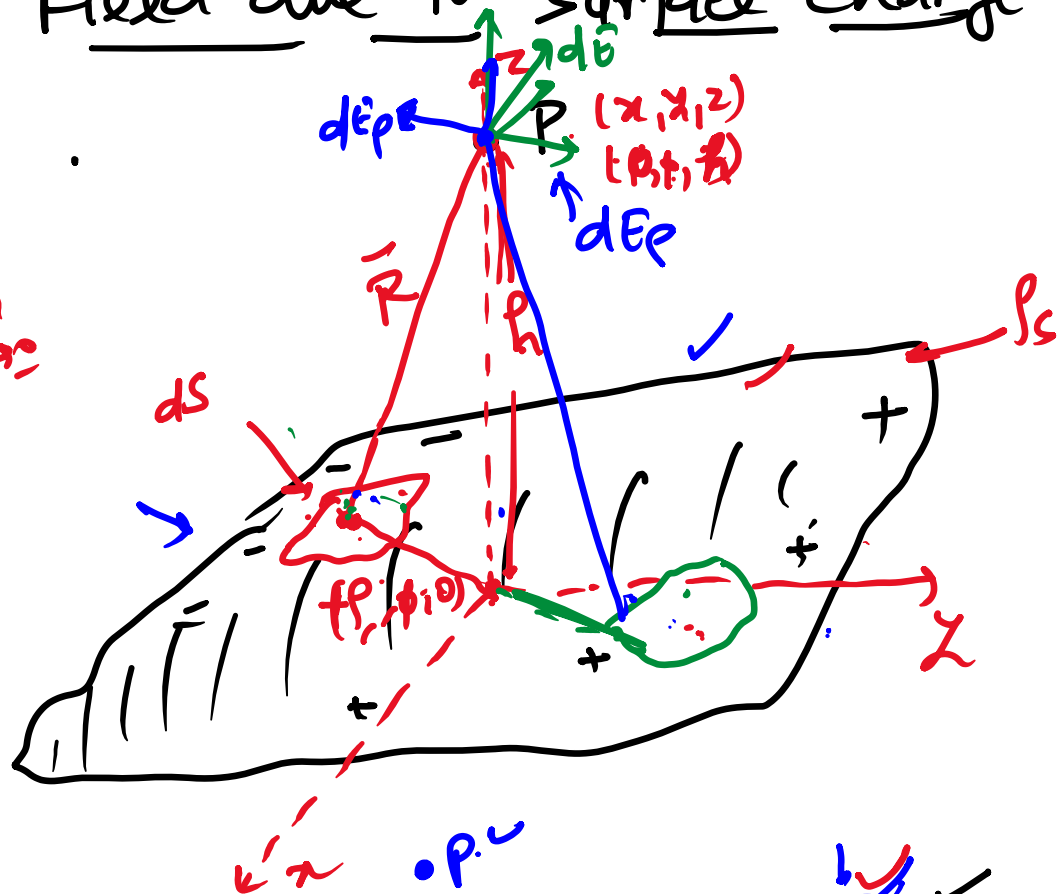
$$A \triangle B \rightarrow -\infty \text{ to } \infty \rightarrow \alpha_1 = -\pi/2, \alpha_2 = \pi/2$$

$$E = \frac{-\rho_L}{4\pi\epsilon_0 r} \int_{-\pi/2}^{\pi/2} (\cancel{\cos\alpha} \hat{r} + \cancel{\sin\alpha} \hat{z}) d\alpha$$

$$E = \frac{\rho_L}{4\pi\epsilon_0 r} [2] \hat{r} = \frac{\rho_L}{2\pi\epsilon_0 r} \hat{r}$$

② Field due to Surface charge -

lim
ds → 0



$$d\vec{E} = \frac{\rho_s}{4\pi\epsilon_0} \frac{\rho d\rho d\phi}{(\rho^2 + h^2)^{3/2}} \left(\frac{-\rho\hat{\rho} + h\hat{z}}{(\rho^2 + h^2)^{3/2}} \right)$$

$$\vec{E} = \frac{\rho_s}{4\pi\epsilon_0} \int_0^\infty \frac{\rho d\rho}{(\rho^2 + h^2)^{3/2}} \int_0^{2\pi} d\phi \Rightarrow \frac{\rho_s}{2\epsilon_0} \hat{z}$$

$$E = \frac{Q}{4\pi\epsilon_0 R^2} \hat{r}$$

$$ds = \rho d\rho d\phi$$

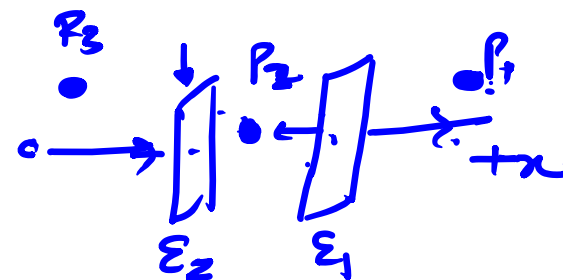
$$Q_s = \rho_s ds$$

$$d\vec{E} = \frac{\rho_s ds}{4\pi\epsilon_0 R^2} \hat{R}$$

$$\vec{R} = -\hat{\rho}\rho + h\hat{z}$$

$$|\vec{R}| = (\rho^2 + h^2)^{1/2}$$

$$\hat{R} = \frac{-\rho\hat{\rho} + h\hat{z}}{(\rho^2 + h^2)^{1/2}}$$



$$P_1 \rightarrow E_1 + E_2$$

$$P_2 \rightarrow E_2 - E_1$$

$$P_3 \rightarrow -E_1 - E_2$$

Q. Calculate the electric field or force due to any charge distribution.

→ Steps →

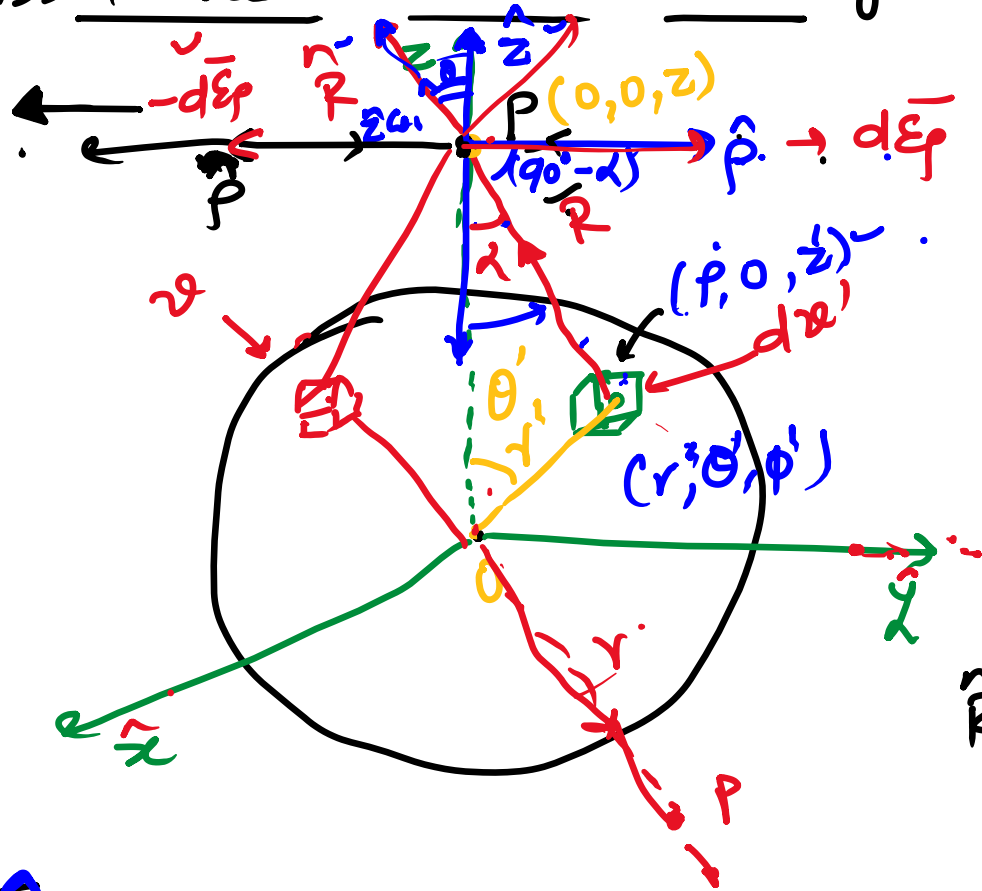


$$\hat{R} = \frac{\vec{R}}{|\vec{R}|}$$

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 R^2} \hat{R}$$

find the position vector $\vec{R}$, the unit vector $\hat{R}$
and calculate $\vec{E}$

Field due to volume charge -



$$Q = \rho_v dV'$$

$$dE = \frac{Q}{4\pi\epsilon_0 R^2} \hat{R}$$

$$d\vec{E} = \frac{\rho_v dV' \hat{R}}{4\pi\epsilon_0 R^2} \quad \text{--- (1)}$$

$$dV' = r'^2 \sin \theta' dr' d\theta' d\phi' \quad \text{--- (2)}$$

$$\hat{R} = \frac{\vec{R}}{|\vec{R}|}$$

$$\vec{R} = \frac{A \quad B}{(x_1, y_1, z_1) \quad (x_2, y_2, z_2)}$$

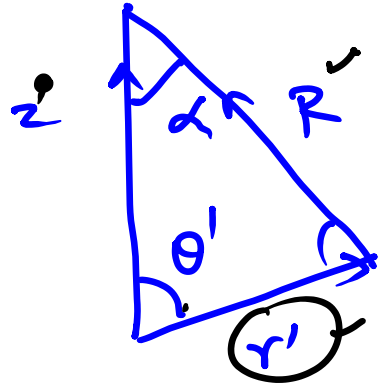
$$\vec{R} = (x_2 - x_1)\hat{x} + (y_2 - y_1)\hat{y} + (z_2 - z_1)\hat{z}$$

$$\hat{R} = \hat{z} \cos \alpha + \hat{p} \sin \alpha$$

$$\vec{R} = \hat{z} \cos \alpha + \hat{p} \sin \alpha$$

$$dE = \frac{\rho_v r'^2 \sin \theta' dr' d\theta' d\phi' \cdot (\hat{z} \cos \alpha + \hat{p} \sin \alpha)}{4\pi\epsilon_0 R^2}$$

Cosine Rule -



$$R^2 = r'^2 + z^2 - 2r'z \cos \theta' \quad \text{--- (3)}$$

$$\rightarrow r'^2 = R^2 + z^2 - 2Rz \cos \alpha \quad \text{--- (4)}$$

$$\text{from (3)} \rightarrow \cos \theta' = -\frac{R^2 - r'^2 - z^2}{2r'z} \quad \text{--- (5)}$$

$$\cos \alpha = -\frac{r'^2 - R^2 - z^2}{2Rz} \quad \text{--- (6)}$$

$$-\sin \theta' d\theta' = -\frac{2RdR}{2r'z} \Rightarrow \sin \theta' d\theta' = \frac{RdR}{r'z}$$

$$dE = \frac{P_v}{4\pi\epsilon_0} \int_{\phi=0}^{2\pi} \int_{r'=0}^R \frac{r',^2}{R^2} dr' \frac{RdR}{r'z} (\cos \alpha \hat{z} + \cancel{\hat{r} \sin \alpha}) d\phi'$$

$$E = \frac{Q}{4\pi\epsilon_0 z^2} \hat{z}$$

$$\Rightarrow E = \frac{Q}{4\pi\epsilon_0 r^2} \hat{r}$$

Similar to Point Charge.

$$\rightarrow E = \frac{Q}{4\pi\epsilon_0 r^2} \hat{r} \rightarrow \text{point charge}$$

$$\lim_{Q \rightarrow 0} \frac{Q}{r^2} \rightarrow \underline{\underline{0}}$$

$$\rightarrow E = \frac{\rho_L}{2\pi\epsilon_0} \hat{\rho} \rightarrow \text{line charge}$$

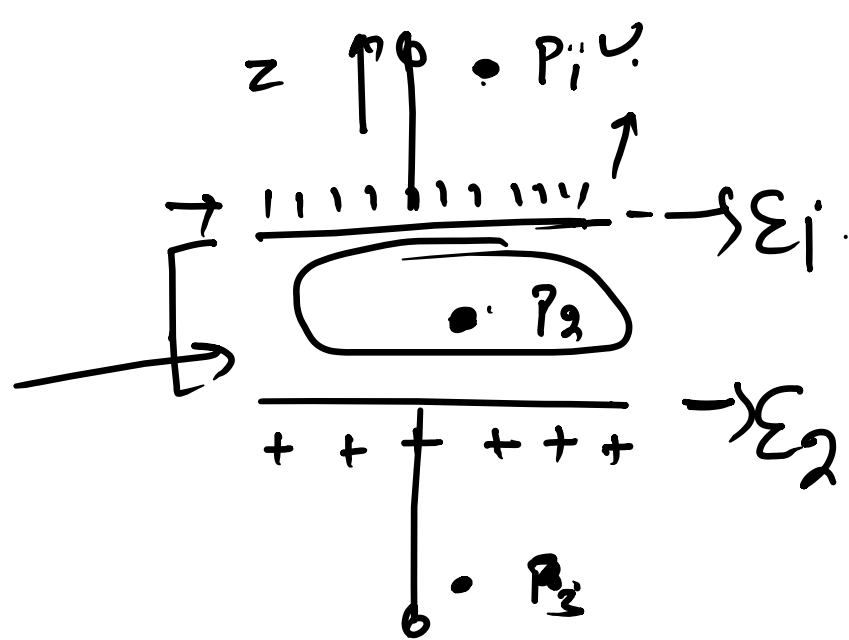
$$\rightarrow E = \left[\frac{\rho_s}{2\epsilon_0} \right] \hat{n} \rightarrow \text{Surface charge}$$

$\hat{n} \rightarrow$ unit vector in the direction of normal to this surface.

$$E = \frac{\rho_v}{4\pi\epsilon_0 r^2} \hat{r} \rightarrow \text{Volume charge}$$

$E \propto \frac{1}{r^2}$	← Point & volume charge
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Solve - Example - 4.4,



$$\textcircled{1} \quad \epsilon_{P_1} = \frac{+\epsilon_1 + \epsilon_2}{2G} \Rightarrow \frac{P_s}{2G} + \frac{P_s}{2G}$$

$$\textcircled{2} \quad \epsilon_{P_2} = \frac{-\epsilon_1 + \epsilon_2}{2G} + \frac{P_s}{2G} = \frac{P_s}{G}$$

$$\textcircled{3} \quad \epsilon_{P_3} = \frac{-\epsilon_1 - \epsilon_2}{2G} = 0$$

Solve - Example - 4.6

Electric flux density $\rightarrow$

Electric displacement -

$$\vec{D} = \epsilon_0 \vec{E}$$

ϵ_0 Dielectric Constant of free space

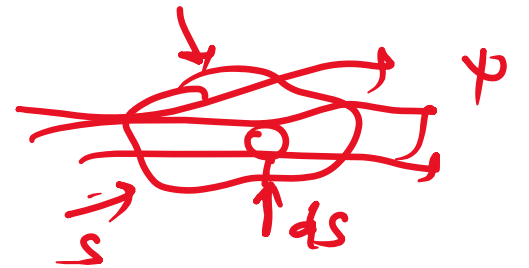
$$\vec{D} = \epsilon_0 \epsilon_r \vec{E}$$

ϵ_r relative permittivity of any medium.

Electric flux $\rightarrow \psi$

$D_c =$ flux per unit area.

$$\vec{D} = \frac{\psi}{ds} \Rightarrow \boxed{\psi = \int \vec{D} \cdot d\vec{s}}$$



Example - 4.7

Gauss's law of Electrostatics -

→ This is one of the Maxwell's equation -

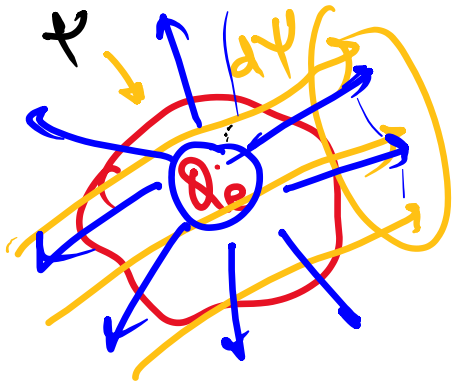
"The net flux through any closed surface is equal to the charge enclosed by that surface!"

Ψ

$$\oint \vec{D} \cdot d\vec{s}$$

Q_{enclosed}

$$\underline{\Psi} = \underline{Q_{\text{enclosed}}}$$



Volume.

$$\nabla \cdot \vec{D} = \rho_v$$

$$D = \epsilon_0 E \Rightarrow \nabla \cdot \vec{E} = \rho_v / \epsilon_0$$

$$\underline{\Psi} = \oint d\Psi = \oint \underline{\vec{D}} \cdot d\vec{s} \quad \text{--- (1)}$$

$$\underline{Q_{\text{enclosed}}} = \int \underline{\rho_v} dv \quad \text{--- (2)}$$

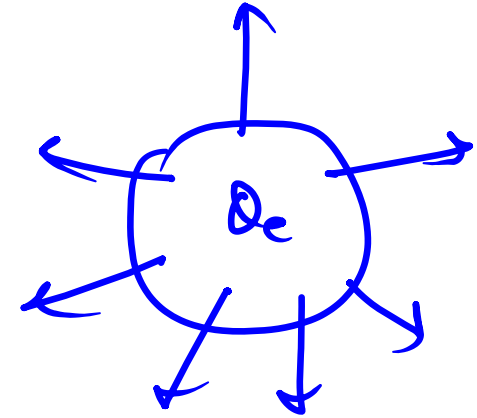
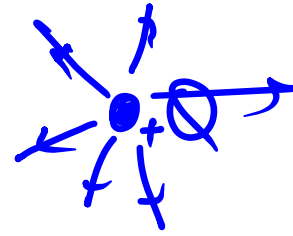
$$\int_V \nabla \cdot \vec{D} dv = \oint \vec{D} \cdot d\vec{s}$$

$$\oint \underline{\vec{D}} \cdot d\vec{s} = \int_V \underline{\rho_v} dv \quad \text{--- (3)}$$

$$\int_V \nabla \cdot \vec{D} dv = \int_V \rho_v dv \quad \text{--- (4)}$$

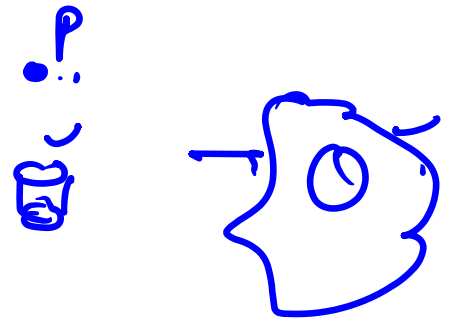
$$\nabla \cdot \vec{D} = \rho_v$$

positive divergence

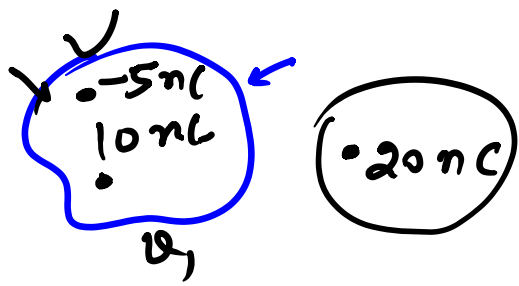


→ Gauss's law can be applied to calculate E or F if charge present in the medium is symmetric.

→ [Application of divergence theorem on
Coulomb's law becomes Gauss's law.]



→ Coulomb's law can be applied on both shapes of charge symmetric or asymmetric for calculation of E/F



→ only charges which are enclosed within the close surface will participate in calculation of E/F .

$$Q_{\text{en}} = 10 - 5 = 5\text{ nC}$$

Applications of Gauss's Law $\rightarrow$

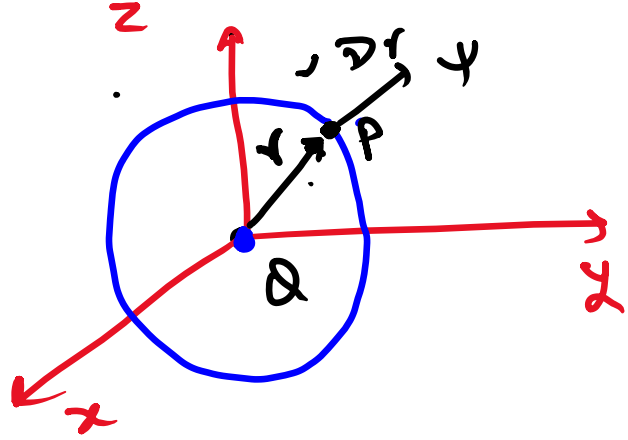
→ ① Assume any Gaussian surface around the charge].

Symmetric

→ (2) Calculate $\psi = \oint \vec{D} \cdot d\vec{s}$ — $\left[\begin{array}{l} \text{normal components} \\ \text{tangential component} \end{array} \right]$.

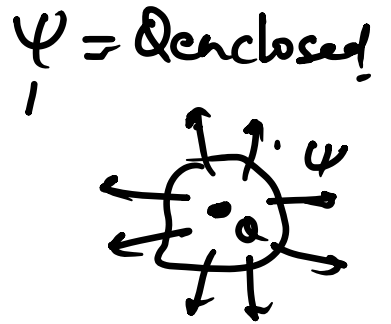
→ ③ $\oint_{\text{tan}} \vec{D} \cdot d\vec{s} = \underline{0}$ & $\oint_{\text{nor}} \vec{D} \cdot d\vec{s} = \underline{\neq 0}$

① Point charge -



$$\Psi = Q_{\text{enclosed}}$$

$$\Psi = \oint \vec{D} \cdot d\vec{S} = D_r \oint d\vec{S} = D_r \int_0^\pi \int_0^{2\pi} r^2 \sin\theta d\theta d\phi$$



$$dS = r^2 \sin\theta d\theta d\phi$$

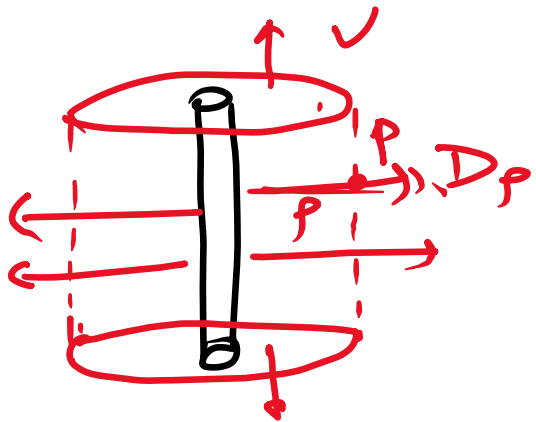
$$\Psi = D_r 4\pi r^2$$

$$D_r = \frac{\Psi}{4\pi r^2} \Rightarrow$$

$$\vec{D} = \frac{\Psi}{4\pi r^2} \vec{r}$$

$$\vec{E} = \frac{\vec{D}}{\epsilon_0} = \frac{\Psi}{4\pi\epsilon_0 r^2} \vec{r} = \frac{Q}{4\pi\epsilon_0 r^2} \vec{r}$$

② Line charge - (infinite)



$$\Psi = Q_{\text{enclosed}}$$

$$\Psi = \oint \vec{D} \cdot d\vec{S} = \underline{\vec{D}_\rho} \oint dS$$

$$dS = \rho d\phi dz$$

$$\underline{\Psi} = D_\rho \int_{\phi=0}^{2\pi} \int_{z=-\infty}^{\infty} \rho d\phi dz = D_\rho 2\pi \rho L$$

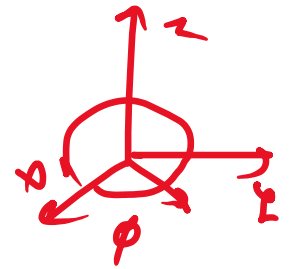
$$\underline{Q} = \rho_L L$$

$$\rho_L L = D_\rho 2\pi \rho L \Rightarrow$$

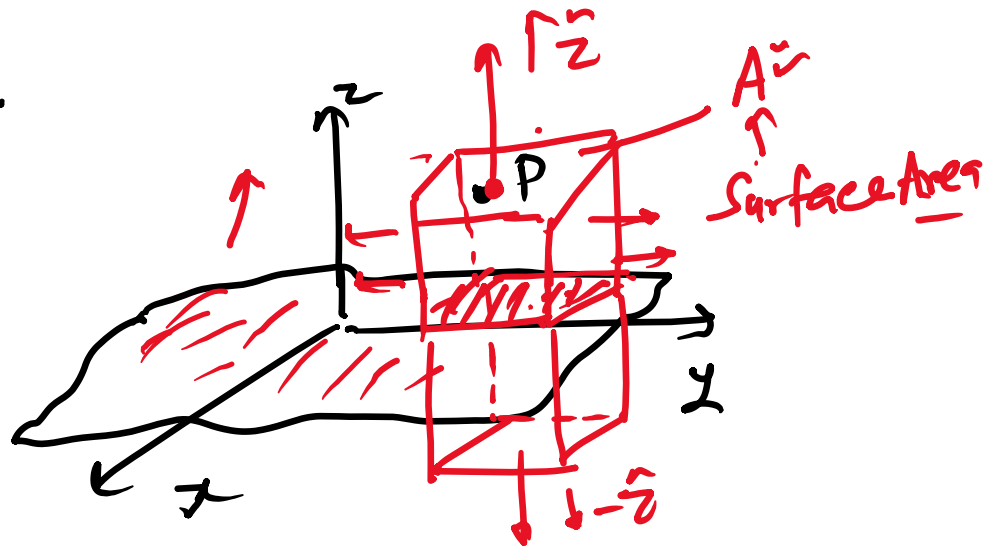
$$D_\rho = \frac{\rho_L}{2\pi \rho}$$

$$\boxed{\vec{D} = \frac{\rho_L}{2\pi \rho} \hat{\rho}}$$

$$\Rightarrow \boxed{\vec{E} = \frac{\rho_L}{2\pi \epsilon \rho} \hat{\rho}}$$



③ Surface charge -



$$\psi = \text{enclosure}$$

$$Q = \rho_s \int dS = \rho_s A \quad \text{--- ①}$$

$$\psi = \oint \vec{D} \cdot d\vec{S} = D_z \left[\int_{\text{top}} dS + \int_{\text{bottom}} dS \right]$$

$$\psi = D_z [A + A] = 2 D_z A \quad \text{--- ②}$$

$$\rho_s A = 2 D_z A \Rightarrow D_z = \frac{\rho_s}{2}$$

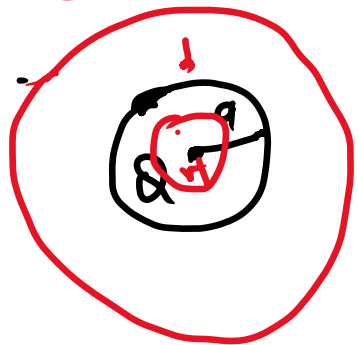
$$\vec{D} = \frac{\rho_s}{2} \hat{z} = \frac{\rho_s}{2} \hat{n}$$

$$\boxed{\vec{E} = \frac{\rho_s}{2\epsilon} \hat{n}}$$

$\hat{n} \rightarrow$ unit normal vector.

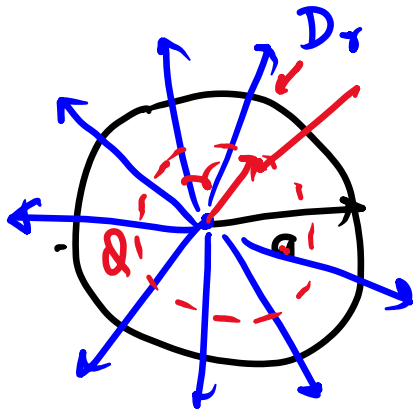
④ volume charge -

Case-I.



$$① r \leq a$$

$$② r \geq a$$



$$\psi = Q_{\text{enclosed}}$$

$$Q = \int_V \rho_v dV = \rho_v \int_0^a \int_0^\pi \int_0^{2\pi} r^2 \sin\theta dr d\theta d\phi$$

$$Q = \rho_v \frac{4}{3} \pi a^3 \quad \text{--- (1)}$$

$$\psi = \oint \vec{D} \cdot d\vec{s} = D_r \oint ds = D_r \int_0^\pi \int_0^{2\pi} r^2 \sin\theta d\theta d\phi$$

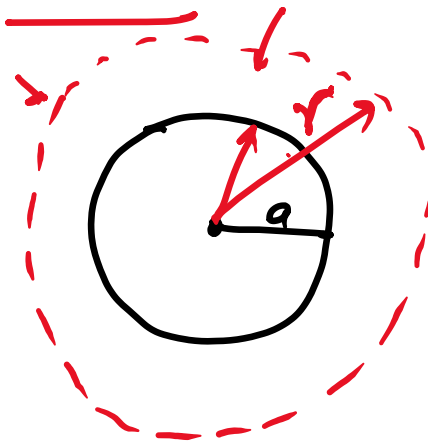
$$\psi = D_r \cdot 4\pi r^2 \quad \text{--- (2)}$$

$$D_r \cdot 4\pi r^2 = \rho_v \frac{4}{3} \pi a^3$$

$$D_r = \frac{\rho_v}{3} r$$

$$\boxed{\vec{D} = \frac{\rho_v r}{3} \cdot \hat{r}}; r \leq a \quad \text{--- (3)}$$

Case-II



$$\psi = Q_{\text{enclosed}}$$

$$Q = \int_V \rho_v dV = \rho_v \int_0^a \int_0^\pi \int_0^{2\pi} r^2 \sin\theta d\theta d\phi dr$$

$$Q = \frac{4}{3} \pi a^3 \rho_v \quad \text{--- (4)}$$

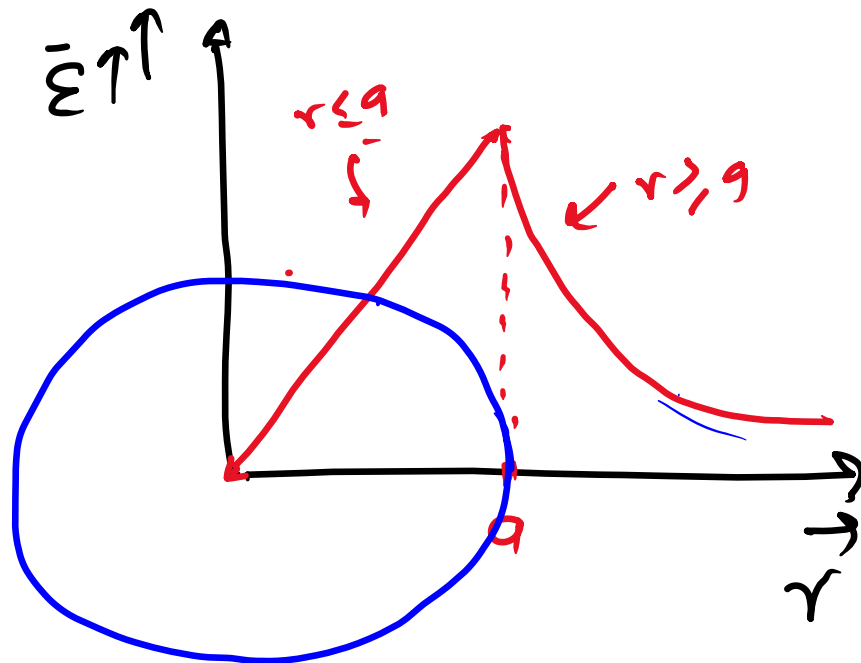
$$\Psi = \oint D ds = D r \int_0^\pi \int_0^\phi r^2 \sin \alpha d\alpha d\phi$$

$$\Psi = D r 4\pi r^2 \text{ --- (5)}$$

$$\frac{4}{3} \pi q^3 \rho_v = D r 4\pi r^2$$

$$D r = \rho_v \frac{q^3}{3r^2} ; r \geq q$$

$$\vec{D} = \begin{cases} \rho_v \frac{r}{3} \hat{r} ; r \leq q \\ \rho_v \frac{q^3}{3r^2} \hat{r} ; r \geq q \end{cases}$$



$$\vec{E} = \rho_v \frac{r}{3\epsilon_0} \rightarrow \underline{2 = \max} ; r \leq q$$

$$\vec{E} \propto \frac{1}{r^2} ; r \geq q$$

① Coulomb's law of forces $\vec{F} = Q \vec{E}$ $\rightarrow \vec{F}, \vec{E}$ Vector $\rightarrow$ Three dimensional information.

② Gauss's law of Electrostatics $\Rightarrow \underline{\psi = Q} \Rightarrow \vec{\nabla} \cdot \vec{D} = \rho \Rightarrow \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$
↑
vector entity

③ Scalar Potential $\rightarrow$

The work done in displacement of any charge from point A to B.

$$dW = \int \vec{F} \cdot d\vec{l}$$

work done by
Some external agent

$$; \quad F = QE$$

$$\Rightarrow dW = -QE \cdot dl$$

$$\Rightarrow W = - \int_A^B QE \cdot dl$$

$$\frac{W}{Q} = - \int_A^B \vec{E} \cdot d\vec{l} = V_{AB} \Rightarrow \text{Potential difference between point A and B.}$$

$$V_{AB} = - \int_A^B \vec{E} \cdot d\vec{l}$$

Point charge

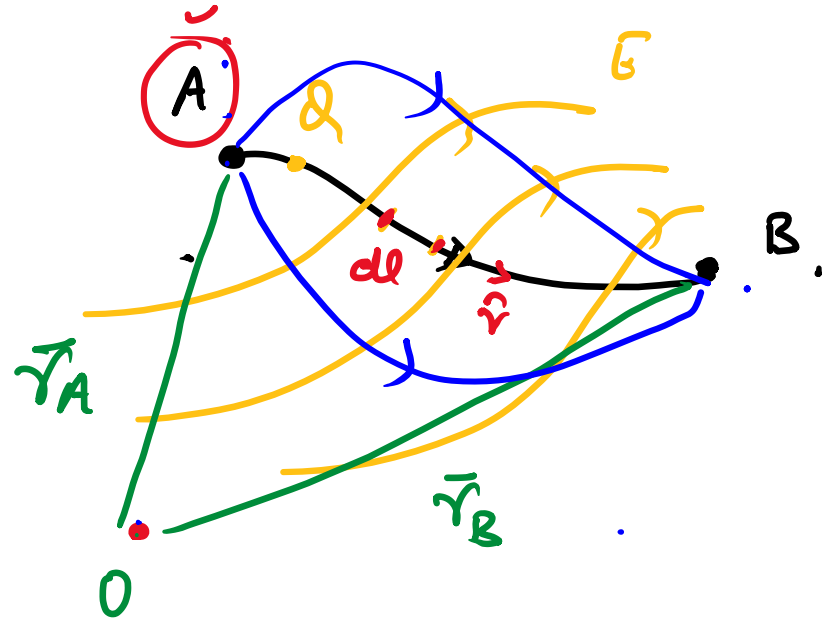
Case-I

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2} \hat{r}$$

$$d\vec{l} = \hat{r} \cdot dr$$

$$V_{AB} = - \int_{r_A}^{r_B} \frac{Q}{4\pi\epsilon_0 r^2} \hat{r} \cdot dr \hat{r}$$

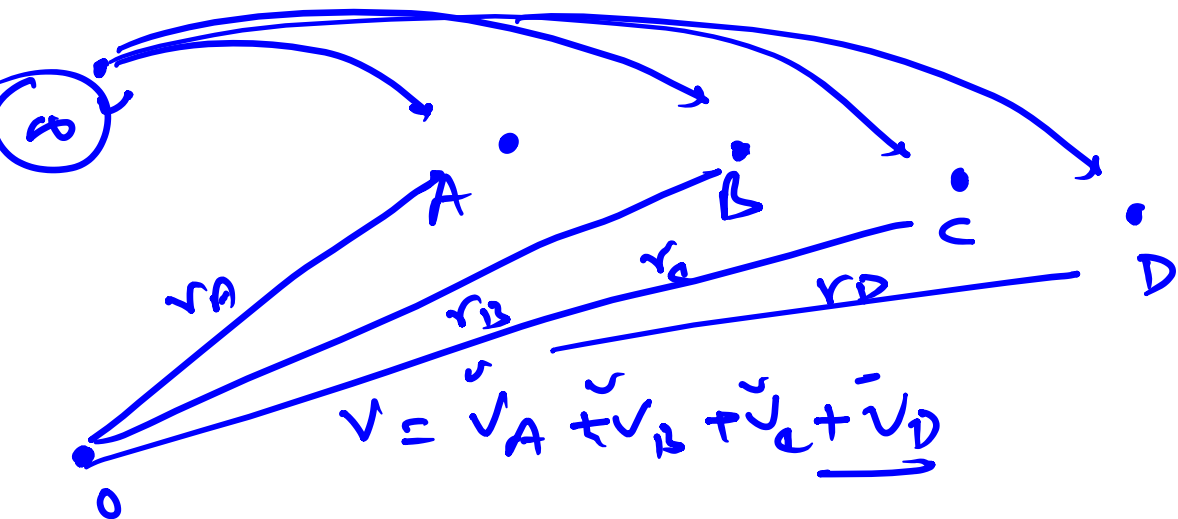
$$V_{AB} = - \int_{r_A}^{r_B} \frac{Q}{4\pi\epsilon_0 r^2} dr = \frac{Q}{4\pi\epsilon_0} \left[\frac{1}{r_B} - \frac{1}{r_A} \right] \Rightarrow \boxed{V_{AB} = V_B - V_A}$$



Case-II If A is placed at infinity.

$$\underline{V_{AB}} = \frac{Q}{4\pi\epsilon_0} \left[\frac{1}{r_B} - 0 \right] = \frac{Q}{4\pi\epsilon_0 r_B} \rightarrow \underline{\text{Potential at point B}}$$

- ① This shows that potential energy is required in displacement of the charge from one point to another. (-ve Sign)
- ② This is independent of path of moving the charge.
- ③ The potential difference is calculated in 'Joules/Coulomb' Volts



$$\vec{V} = \int \frac{\rho_v(r') dl'}{4\pi\epsilon_0 |r-r'|}$$

$$V = \int \frac{\rho_s(r') dS'}{4\pi\epsilon_0 |r-r'|}$$

$$V = \int \frac{\rho_v \rho_v}{4\pi\epsilon_0 |r-r'|}$$

Relationship between E & V

① → potential is independent on Path

$V_{AB} = -V_{BA}$

$\Rightarrow V_{AB} + V_{BA} = 0$

$\int \uparrow$

Total potential in movement of charge on a circular path.

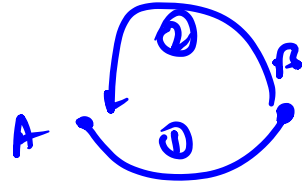
$\oint \vec{E} \cdot d\vec{l} = 0$

→ Conservative field.

→ Conservative field → no loss of potential energy in movement of charge



→ no displacement of charge. over a circular path.



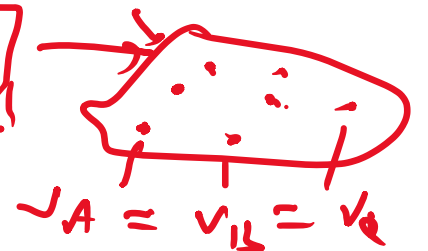
→ Stoke's theorem - $\oint \vec{E} \cdot d\vec{l} = \int_S \nabla \times \vec{E} \cdot d\vec{S} = 0$

Note:- for time varying field this equation is modified and called Faraday's law

$$\nabla \times \vec{E} = 0$$

irrotational, conservative,
→ maxwell's equation
for static electric field

Equipotential surface



$$\nabla \times \mathbf{E} = 0$$

$\nabla \times \nabla V \rightarrow$ Carl of gradient of any scalar field is zero.

$$\underline{\nabla \times \nabla V = 0}$$

$$\nabla \times \mathbf{E} = 0$$

$$\Rightarrow \nabla \times \nabla V = 0$$

$$\boxed{\mathbf{E} = -\nabla V} \checkmark$$

$$\rightarrow \text{chain rule} \cdot dV = \frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz$$

$$= \left(\hat{x} \frac{\partial V}{\partial x} + \hat{y} \frac{\partial V}{\partial y} + \hat{z} \frac{\partial V}{\partial z} \right) (\hat{x} dx + \hat{y} dy + \hat{z} dz)$$

$$-\mathbf{E} = \nabla V \Rightarrow \boxed{\mathbf{E} = -\nabla V}$$

$$V = - \int \mathbf{E} \cdot d\mathbf{l}$$

$$dV = - \mathbf{E} \cdot d\mathbf{l}$$

$$\mathbf{E} \cdot d\mathbf{l} = (\hat{x} E_x + \hat{y} E_y + \hat{z} E_z) (\hat{x} dx + \hat{y} dy + \hat{z} dz) \text{ --- (1)}$$

Scalar Potential

Q Given $V = \frac{10}{r^2} \sin \theta \cos \phi$

- ① find electric flux density $\vec{D}$ at $(2, \pi/2, 0)$
- ② Calculate the work done in moving charge $10 \mu\text{C}$ from point $A(1, 30^\circ, 120^\circ)$ to $B(4, 90^\circ, 60^\circ)$

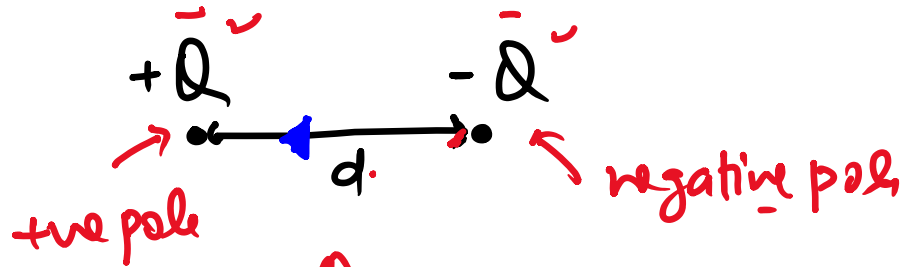
Solⁿ

~~$\nabla \cdot \vec{D} = \rho$~~
 ~~$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$~~
 ~~$\vec{D} = \epsilon_0 \vec{E}$~~

$\vec{D} = -\epsilon_0 \nabla V$

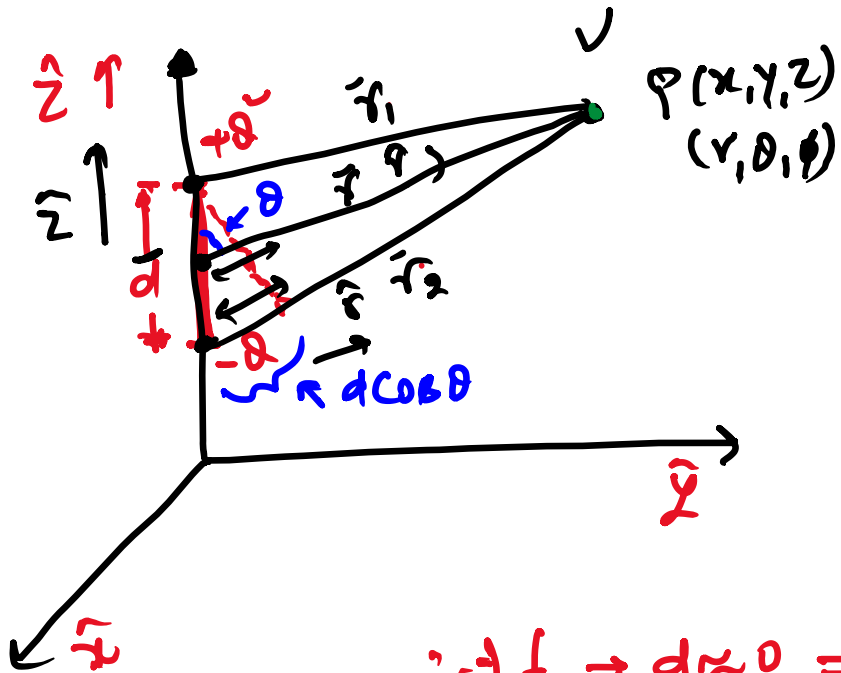
An electric dipole & flux lines →

Dipole-



dipole moment → (from $-Q$ to $+Q$)

$$\vec{p} = Q\vec{d}$$



$d \rightarrow$ very small $\rightarrow$ infinitesimal

$$V = \frac{+Q}{4\pi\epsilon_0 r_1} + \frac{-Q}{4\pi\epsilon_0 r_2}$$

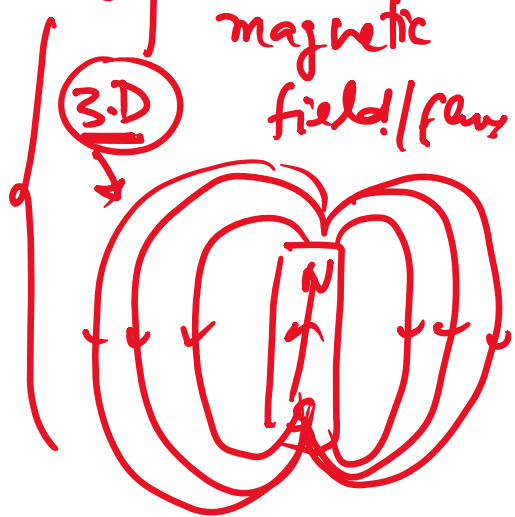
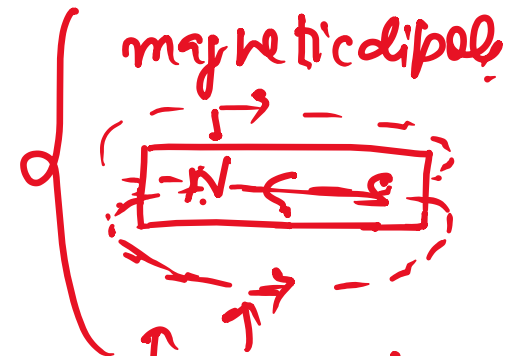
$$V = \frac{Q}{4\pi\epsilon_0} \left[\frac{1}{r_1} - \frac{1}{r_2} \right] = \frac{Q}{4\pi\epsilon_0} \left[\frac{r_2 - r_1}{r_1 r_2} \right]$$

$$\rightarrow f \rightarrow d \approx 0 \Rightarrow r_1 \approx r_2 \approx r, \quad r_2 - r_1 = d \cos \theta$$

$$\odot \rightarrow \vec{p}$$

$$\vec{E}, \odot$$

Dipole → Resonance
Antenna, filter and many more
Field distribution



$$\rightarrow V = \frac{Q}{4\pi\epsilon_0} \frac{d \cos \theta}{r^2} \quad \text{--- (1)}$$

$$d \cos \theta = |\underline{d}| |\underline{\hat{r}}| \cos \theta$$

$$d \cos \theta = \underline{d} \cdot \underline{\hat{r}}$$

$$V = \frac{Q}{4\pi\epsilon_0} \frac{\underline{d} \cdot \underline{\hat{r}}}{r^2} \quad \text{--- (2)}$$

$$\underline{d} = |\underline{d}| \underline{\hat{z}} = d \underline{\hat{z}} \rightarrow$$

$$\underline{P} = Q \underline{d} = Q d \underline{\hat{z}} \leftarrow \text{dipole moment}$$

$$\rightarrow V = \frac{\underline{P} \cdot \underline{\hat{r}}}{4\pi\epsilon_0 r^2} \quad \text{--- (3)}$$

if dipole is present at position $\underline{r}'$ $\rightarrow$

$$\rightarrow V = \frac{\underline{P} \cdot (\underline{r} - \underline{r}')}{4\pi\epsilon_0 |\underline{r} - \underline{r}'|^3} \quad \text{--- (4)}$$

Calculate the electric field -

$$\underline{E} = -\nabla V$$

$$= - \left[\underline{\hat{r}} \frac{\partial V}{\partial r} + \frac{\hat{\theta}}{r} \frac{\partial V}{\partial \theta} \right]$$

$$\underline{E} = - \frac{\underline{\hat{r}}}{4\pi\epsilon_0} \frac{\partial d \cos \theta}{\partial r} \frac{\partial}{\partial r} \left(\frac{1}{r^2} \right) - \frac{\hat{\theta}}{r} \frac{Q d}{4\pi\epsilon_0 r^2} \frac{\partial \cos \theta}{\partial \theta}$$

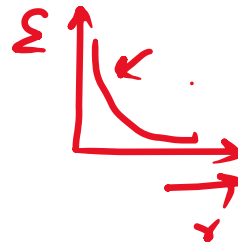
$$\underline{E} = \frac{\underline{P}}{4\pi\epsilon_0 r^3} (2 \cos \theta \underline{\hat{r}} + \sin \theta \underline{\hat{\theta}})$$

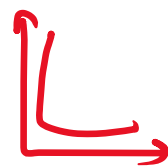
Point charge ✓

Dipole ✓

Quadrupole ✓
(combination of two dipoles)

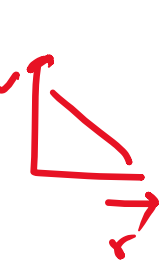
Sextupole ✓
(combination of two Quadrupole)

$$\frac{\vec{E}}{\propto \frac{1}{r^2}}$$


$$\propto \frac{1}{r^3}$$


$$\propto \frac{1}{r^4}$$

$$\propto \frac{1}{r^5}$$

$$\propto \frac{1}{r}$$


$$\propto \frac{1}{r^2}$$

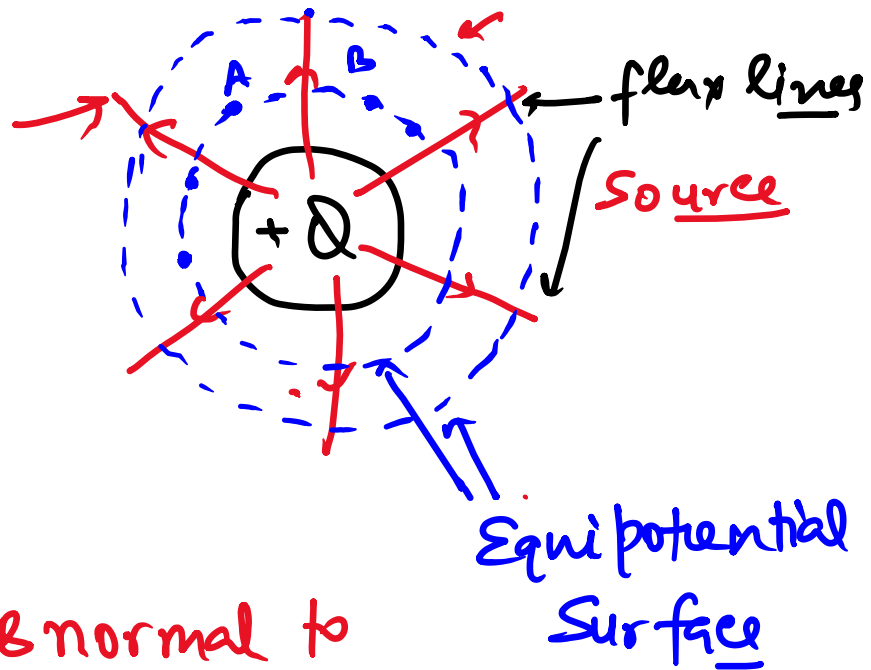
$$\propto \frac{1}{r^3}$$

$$\propto \frac{1}{r^4}$$

Flux lines $\rightarrow$
Point charge

Equipotential Surface

$$\left\{ \begin{array}{l} V_{AB} + V_{BA} = 0 \rightarrow \underline{\epsilon PS} \\ -\oint \vec{E} \cdot d\vec{l} = 0 \rightarrow \text{conservative} \\ \nabla \times \vec{E} = 0 \rightarrow \text{irrotational} \end{array} \right.$$



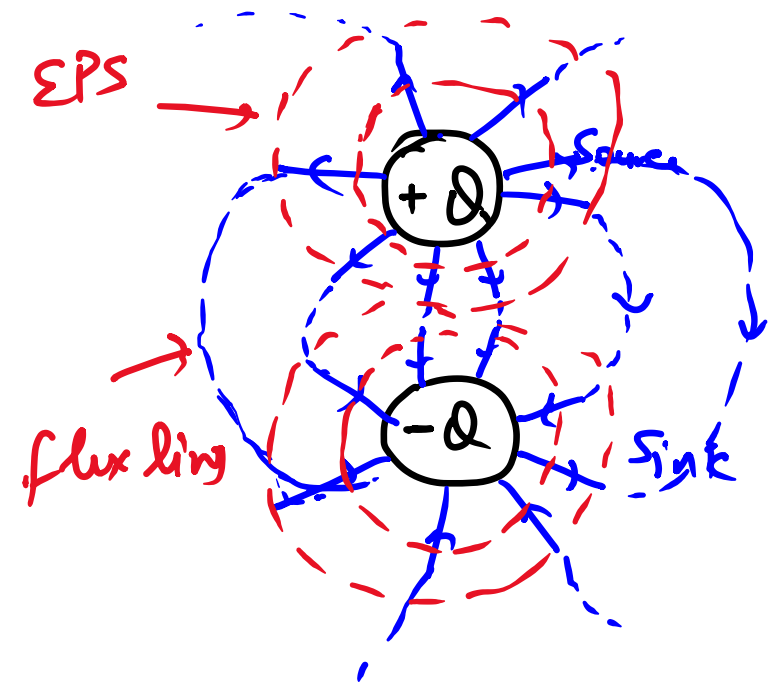
$\vec{D} \rightarrow$ flux density is tangential to flux lines.

$$\overline{V_A - V_B} = 0$$

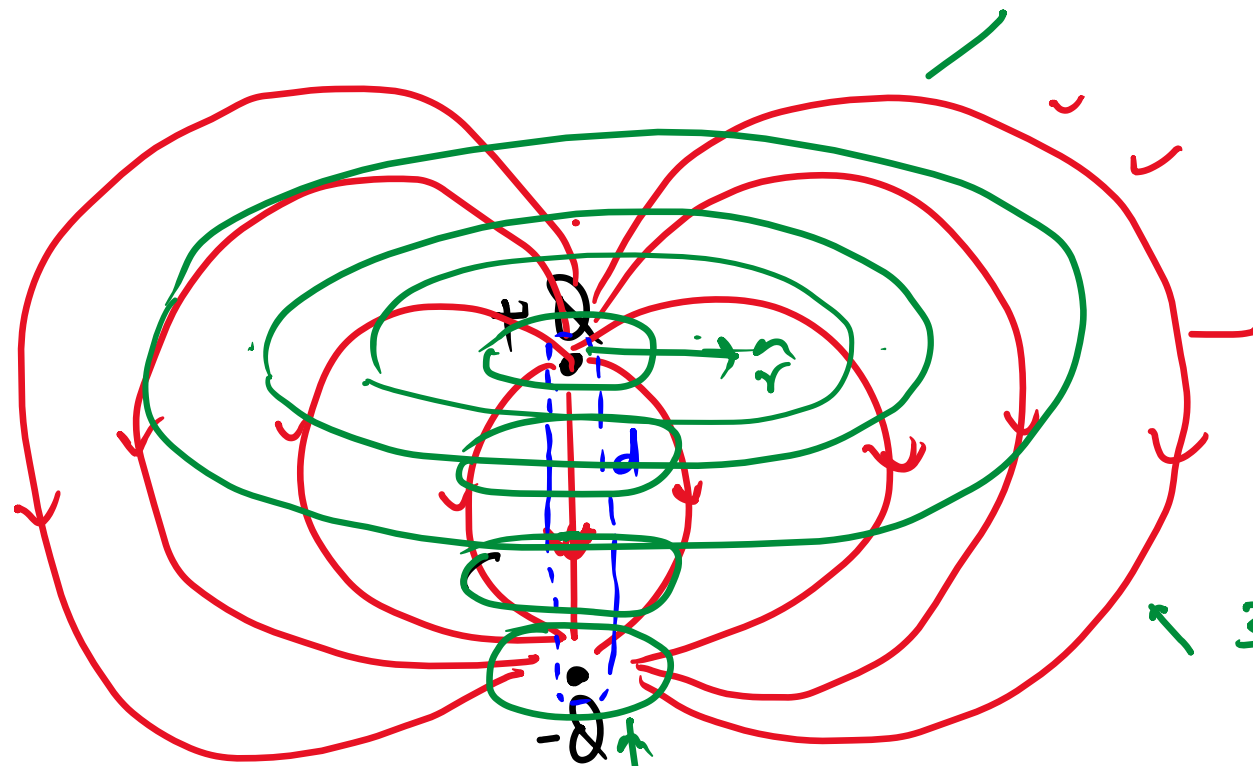
$$\oint \vec{E} \cdot d\vec{l} = 0$$

① $\vec{D}$ is normal to the EPS.

② $\vec{D}$ is tangential to flux lines.



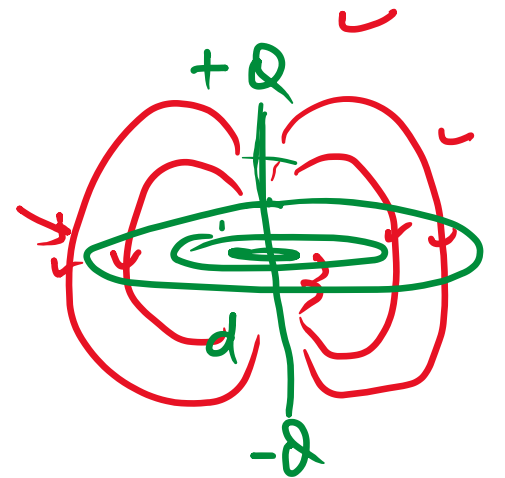
$$\boxed{\nabla \times \underline{E} = 0} \quad \oint \underline{E} \cdot d\underline{l} = 0$$



Electric flux lines

3 Dimensional

magnetic field distribution



$$\underline{H} = |\underline{H}| \cdot \hat{r}$$

$$\underline{E} = |\underline{E}| \cdot \hat{\theta}$$

Q Two dipoles with dipole moments $-5\hat{z}$ nC/m & $9\hat{z}$ nC/m are located at points $(0, 0, -2)$ & $(0, 0, 3)$, respectively. Find the potential at the origin.

Sol $\rightarrow$

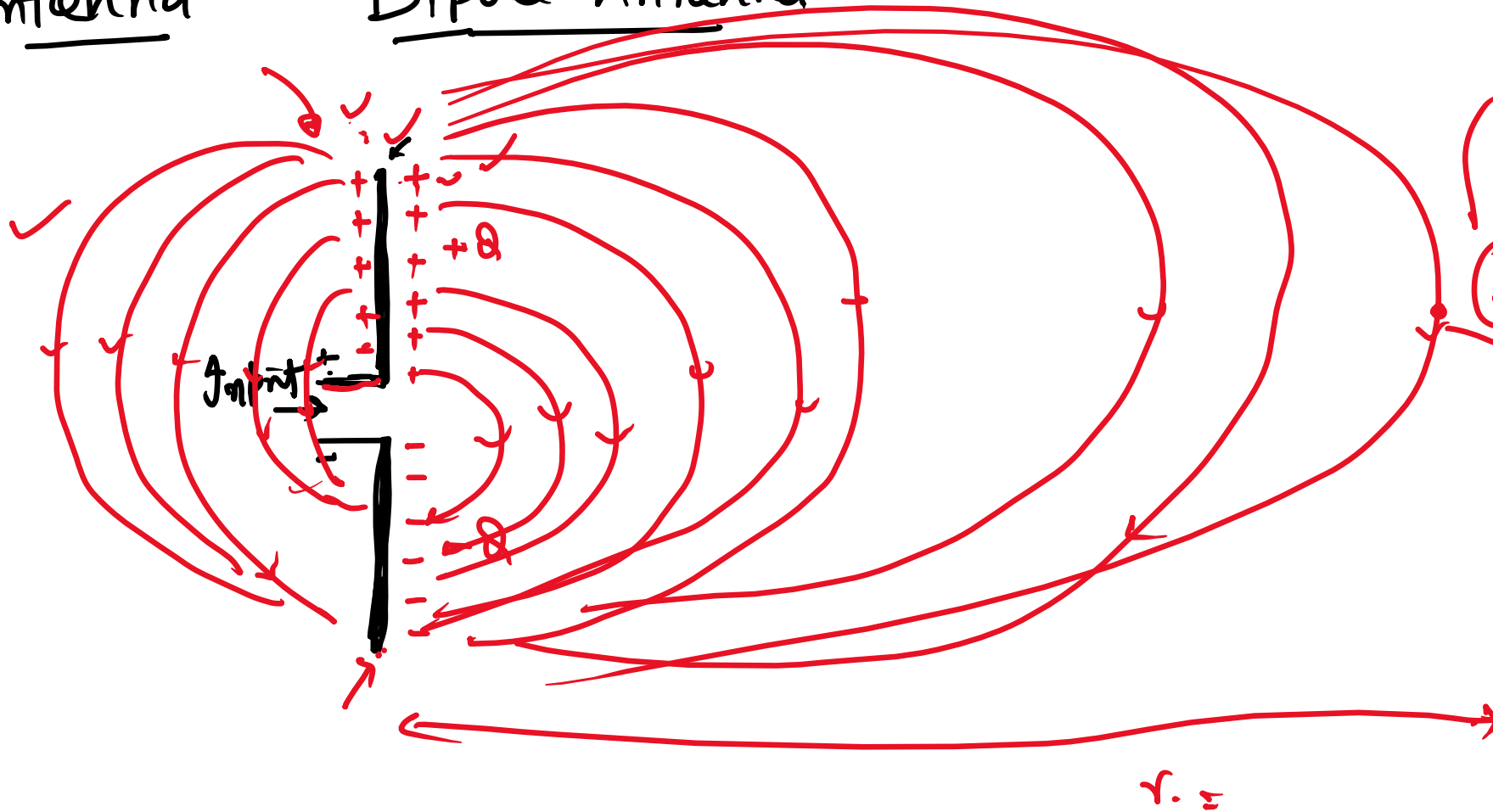
$$\vec{P}_1 = -5\hat{z} \text{ nC/m} \rightarrow (0, 0, -2)$$

$$\vec{P}_2 = 9\hat{z} \text{ nC/m} \rightarrow (0, 0, 3)$$

$$V = \frac{\vec{P}_1 \cdot (\vec{r} - \vec{r}'_1)}{4\pi\epsilon_0 |\vec{r} - \vec{r}'_1|^3} + \frac{\vec{P}_2 \cdot (\vec{r} - \vec{r}'_2)}{4\pi\epsilon_0 |\vec{r} - \vec{r}'_2|^3}$$

Antenna-

Dipole Antenna



$$P = Q(d)$$

$$V = \frac{\vec{p} \cdot \vec{r}}{4\pi\epsilon_0 r^2}$$

$$\vec{E} = -\nabla V$$

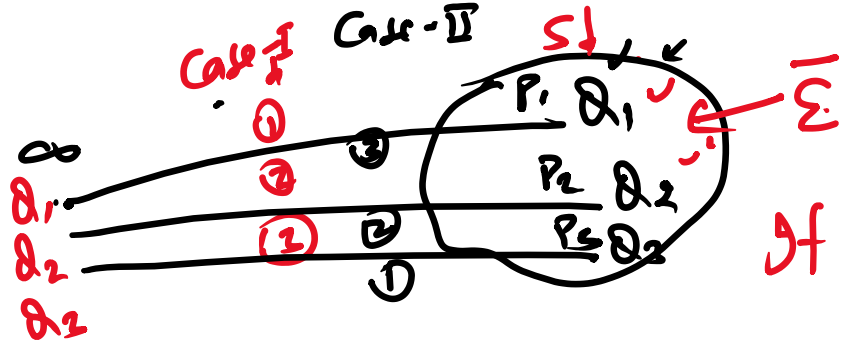
$$\vec{H} = \frac{1}{4\pi\epsilon_0} \frac{\vec{p} \times \vec{r}}{r^3}$$

Energy density in electrostatic fields -

Case-I $\rightarrow \underline{W} = \underline{W}_1 + \underline{W}_2 + \underline{W}_3$

$$W = 0 + Q_2(V_{21}) + Q_3(V_{31} + V_{32}) \quad \text{--- (1)}$$

$$\begin{cases} V = W/Q \\ \Rightarrow W = VQ \end{cases}$$



If charges are placed in reverse order

$W_1 = 0$, because there is no electric field in space's

Case-II

$$W = W_3 + W_2 + W_1$$

$$W = 0 + Q_2(V_{23}) + Q_3(V_{32} + V_{31}) \quad \text{--- (2)}$$

$$[V_{21} = V_1 - V_2]$$

$$[V_{31} = V_1 - V_3]$$

$$[V_{32} = V_2 - V_3]$$

add eqn (1) & (2) -

$$2W = Q_1(\underline{V_{12}} + \underline{V_{13}}) + Q_2(\underline{V_{21}} + \underline{V_{23}}) + Q_3(\underline{V_{31}} + \underline{V_{32}})$$

$$2W = Q_1 V_1 + Q_2 V_2 + Q_3 V_3$$

$$\underline{W = \frac{1}{2} (Q_1 V_1 + Q_2 V_2 + Q_3 V_3)} \quad \text{--- (3)}$$

$$\begin{cases} V_{12} = V_2 - V_1 \\ V_{13} = V_3 - V_1 \end{cases}$$

$$W = \frac{1}{2} \sum_{k=1}^n Q_k V_k \quad \text{--- (5)}$$

← This is the case for the displacement of point charges.

$$\int_S \underline{\vec{A}} \cdot d\underline{\vec{s}} = \int_V \underline{\nabla} \cdot \underline{\vec{A}} dV$$

→ In the case of Continuous charge -

$$W = \frac{1}{2} \int_L \rho_L V dL \quad ; \quad Q = \int_L \rho_L dL$$

$$W = \frac{1}{2} \int_S \rho_S V dS \quad ; \quad Q = \int_S \rho_S dS$$

$$W = \frac{1}{2} \int_V \rho_V V dV \quad ; \quad Q = \int_V \rho_V dV$$

for volume charge → $\rho_V = \underline{\nabla} \cdot \underline{\vec{D}} \rightarrow$ Gauss's law of electrostatics ✓

$$W = \frac{1}{2} \int_V (\underline{\nabla} \cdot \underline{\vec{D}}) \underline{V} dV$$

$$\underline{V}(\underline{\nabla} \cdot \underline{\vec{D}}) = [\underline{\nabla} \cdot (\underline{V} \underline{\vec{D}}) - \underline{\vec{D}} \cdot (\underline{\nabla} \underline{V})] \quad \checkmark$$

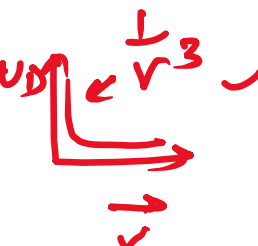
$$\left[\underline{\nabla} \cdot (\underline{V} \underline{\vec{A}}) = \underbrace{\underline{\vec{A}} \cdot \underline{\nabla} \underline{V}}_{\substack{\rightarrow \text{vector} \\ \text{Scalar}}} + \underbrace{\underline{V} \cdot (\underline{\nabla} \cdot \underline{\vec{A}})}_{\substack{\text{Scalar} \\ \leftarrow}} \right]$$

$$W = \frac{1}{2} \left[\int_V (\nabla \cdot (v \bar{D}) - \bar{D} \cdot (\nabla v)) dV \right]$$

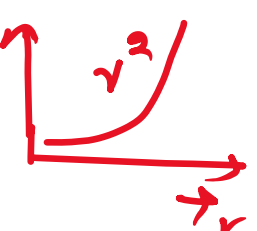
$$W = \frac{1}{2} \int_V \nabla \cdot (v \bar{D}) dV - \frac{1}{2} \int_V \bar{D} (\nabla v) dV$$

$$W = \left[\frac{1}{2} \oint_S (v \bar{D}) dS \right] - \frac{1}{2} \int_V \bar{D} (\nabla v) dV$$

① $v \propto \frac{1}{r}, \bar{D} \propto \frac{1}{r^2} \Rightarrow (v \bar{D}) \propto \frac{1}{r^3}$



② $dS \propto r^2 ds$



→ $dS \rightarrow$ becomes very large
 → $v \bar{D} \rightarrow$ becoming very small
 } → net effect of first term is $\approx$ zero.

$$W = - \frac{1}{2} \int_V \bar{D} (\nabla v) dV \quad \text{--- (C) } \Rightarrow \underline{\underline{\bar{\Sigma} = -\nabla v}}$$

$$\begin{cases} \bar{D} = \frac{Q}{4\pi r^2} \hat{r} \\ v = \frac{Q}{4\pi \epsilon_0 r} \end{cases}$$

$$W = -\frac{1}{2} \int_V \nabla \cdot (-\vec{E}) dV = \frac{1}{2} \int_V \vec{D} \cdot \vec{E} dV \Rightarrow \vec{D} = \epsilon_0 \vec{E}$$

$$\vec{E} \cdot \vec{E} = E^2$$

$$W = \frac{1}{2} \int_V \epsilon_0 E^2 dV \quad \text{--- (7)}$$

Energy density -

$$w = \frac{dW}{dV}$$

$$w = \frac{dW}{dV} = \frac{1}{2} \epsilon_0 E^2 \Rightarrow$$

$$w = \frac{1}{2} \epsilon_0 E^2$$

Electrostatic field

